

## Overview of School Assignment Algorithms

Parag Pathak

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Introductions

Matching Markets

Algorithm: Who, what, why?

Good properties

DA/IA/TTC

Q&A and Discussion

Please ask questions throughout in the chat, as well

## Agenda

### Introductions

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## WE ARE FIGHTING SYSTEMIC INEQUALITY IN EDUCATION







#### Efficient

Avela's tools are easy to use, reduce costs, and save time.

### Equitable

Our tools give all families an equal opportunity and reduce bias in processes.

### Empowering

We power transparent, auditable processes that increase trust and agency.

## **TEACHFOR AMERICA**

Department of



Corps Member satisfaction and retention increased 33 and 16 percent 30,000 more students were able to choose their preferred school

**Education** 

Student achievement increased 0.4 and 0.14 standard deviations in math and reading

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Avela is your partner for equity in education across the admissions, enrollment, and funding pipeline.

Family Communication and Information

Application Processes Admissions Policies and Enrollment Lotteries

Funding Allocations

#### Avela Explore

Mobile-first guide empowers candidates to find the best schools and programs.



#### **Avela Apply**

Accessible online application platform that ensures successful submissions. (Coming soon)

#### Family Tools

# Avela Select

Intuitive tool for equitable selection of cohorts and creation of admission policies. (Coming soon)

#### Avela Match

Administrator Tools

Powerful matchmaking platform to centrally allocate individuals across opportunities.

### Avela Explore <sup>™</sup> - School finder and enrollment guide

- Customized To Your Community
- Accessible
  - Mobile Optimized
  - o Multilingual
  - ADA Compliant
- Dynamic Updating
  - Filters, No Wizard!
  - Instant Feedback/Results
- Recommendations
  - Encourages Ranking
  - Eligibility, Neighborhoods
  - Commute Distance
  - o Similar Schools
- School Information
  - Customizable Profile
  - Automatically updated



### Avela Match<sup>™</sup> - Centralized school assignment

#### • Modern and Delightful

- Cloud-hosted, web-based
- Drag-and-Drop Interface
- Easy Configuration
- Powerful and Flexible
  - Supports wide range of assignment policies and placement algorithms
  - Match on any data field
- Forecast and Plan
  - Experiment & Run Scenarios
- Easily Upload Data
  - Excel/CSV Files
  - Salesforce and SIS Integration Coming soon





### Built on Nobel Prize Winning Science

Avela's software is built on sophisticated matchmaking algorithms pioneered by David Gale, Tayfun Sönmez, Lloyd Shapley, and Al Roth.

### Enrollment Design Experts

Avela was co-founded by Parag Pathak and Joshua Angrist, Professors of Economics at MIT and leaders in the field of market and enrollment design.

#### **Parag Pathak**

Co-Founder & Chief Economist, Avela Class of 1922 Professor of Economics, MIT

- Co-Founder, School Effectiveness & Inequality Initiative
- Co-Founder, NBER Working Group on Market Design
- Advisor to School Districts, Nonprofits, and US Army
- John Bates Clark Medal Recipient
- PhD, Harvard; AB, Harvard

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## POLL



## What's your primary role?

- 1. District/CMO Administrator
- 2. Teacher or School Administrator
- 3. Researcher / Professor
- 4. Advocate / Nonprofit / Foundation
- 5. Consultant
- 6. Other



Introductions

## **Matching Markets**

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## **Matching markets**



- In many situations, when something is scarce, individuals who are willing to pay the most, get what they want
  - If you are trying to buy a house, you can be outbid by someone else
- But for many of life's most important transactions, money isn't everything
- Not only do you chose, but you have to be chosen
  - Getting into college: I can't simply chose to go to MIT, MIT also has to pick me
  - Choosing a spouse
  - Getting a kidney donation
- These situations are known as **matching markets**

## Matching markets



- In many markets, the price equilibrates
- The price is the level at which demand equals supply
  - If demand > supply, the price increases
  - If demand < supply, the price decreases
- Without prices, we have to think more carefully how the market equilibrates
  - What is the supply? What is the demand?
  - An algorithm helps to match supply and demand
- In K-12, the supply = school seats, the demand = what families want, and in a choice system, an algorithm equilibrates demand and supply
  - What if there is no choice system? What does the equilibration? The housing market!



Introductions

Matching Markets

## Algorithm: Who, what, why?

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Q&A and Discussion

#### What is an Algorithm?





Output



#### Algorithm is part of larger school choice ecosystem



#### • Key to a good algorithm are the inputs

- Today, we will focus on algorithm with inputs as given
- But this doesn't mean these aspects aren't central....

#### • Helping families form preferences

- Decision aids
- School finders

#### • How to decide on school priorities

- Walk zone/Sibling/Diversity
- Also: School portfolio and programming, zone definitions, transportation policies, and other rules
  - We will talk about these issues at other webinars



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Q&A and Discussion

#### What makes a good algorithm?



- Algorithmic questions:
  - Can you make it easy for families to rank schools?
  - Can you give as many families as possible their top choice?
  - Can you make sure no families can complain about their assignment and be justified?
- Unfortunately, it is mathematically impossible to say yes to all of these questions
- Process questions:
  - Can you explain your process to participants?
  - Can you give advice to participants?
  - Can you replicate or audit your algorithm?
  - Can you use your process to drive continuous improvement?
- Let's start with the first three questions

#### **Property #1: Honesty is best policy**



- After a family has determined which schools they like, they tell the assignment system
- In a single-best offer system, a family must submit a preference rank
- Many families wonder if they should...
  - Avoid ranking schools that others want
  - Rank schools where they obtain priority
- A assignment mechanism where students do not benefit from reporting dishonestly is called a **strategy-proof** system
- To explain why this is a desirable property, we describe another system that is not

#### **Case study 1: Immediate Acceptance in Boston**

## School assignment flaws detailed

#### Two economists study problem, offer relief

#### By Gareth Cook GLOBE STAFF

Boston uses a deeply flawed system for assigning students to its public schools, pushing more students out of their top-choice schools than necessary and giving parents a reason to lie about which schools they want, according to a pair of researchers who recently published their findings in a leading economics journal. A new system, they say, could greatly reduce the anxiety in the city's annual school-choice process, in which thousands of parents submit lists of their top choices and await the computer-generated decision that will affect the next year to five years of their child's education.

The researchers found that once the parents submit their lists, they are subject to a poorly designed method of allocating spots in the top schools. By using a different technique, they say, the city could get more students into one of their top-choice schools while also making the system fairer. The alternate technique, which the researchers outline in the paper, could be put in place with relatively simple, inexpensive changes and would not require the city to change any of its broader policies, according to the researchers and other academics who have seen the paper.

"Once all this is known, I don't see how they can keep the Boston mechanism," said Turkish economist Tayfun Sonmez, one of the researchers who studied Boston's system.

For more than two decades, policymakers have devoted enormous amounts of attention to various ways to assign students to schools, sparking philosophical debates, charges of racial and economic discrimination, and tangled court battles — all of which have played out with particular drama in Boston. But the authors say their work, which also examined districts in Columbus, Minneapolis, and Seattle, is the first rigorous examination of how best to do the actual matching once the policy is decided.

The research has broader implications as well. If more parents were happier with their school assignments, it would help keep them from fleeing for the suburbs and bolster the fortunes of the school district — and the city. Officials with the Boston public schools and the Boston School Committee readily acknowledge that parents are frustrated with



#### **Immediate Acceptance**



• **Round 1**: In Round 1, only the first choices of the students are considered. For each school, consider the students who have listed it as their **first choice** and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her first choice.

• **Round k**: Consider the remaining students. In Round k **consider only the k choices** of these students.. For each school with still available seats, consider the students who have listed it as their k choice and assign the remaining seats to these students one at a time following their priority order who has listed it as her kth choice until either there are no seats left or there is no student left ho has listed it as her kth choice.

#### Example for Estella choosing schools

	Choice	Sibling	Walk-Zone	Random #
Murphy	1	No	No	5
Channing	2	Yes	No	5
Perkins	3	No	Yes	5

- At the Murphy, three categories: Sibling-Walk, Sibling, and Walk come before Estella's group.
- At Channing, Estella has Sibling priority so she is likely to get an offer there
- **Poll:** Which school should Estella rank first? Channing, Murphy, or Perkins?



#### Immediate Acceptance: How to Participate?



- St. Petersburg Times (2007)
  - Make a realistic, informed selected on the school you list as your first choice. It's the cleanest shot you will get at a school. It's the cleanest shot you will get at a school, but if you aim too high you might miss.

Here's why: If the random computer selection rejects, your first choice, your chances of getting your second choice schools are greatly diminished. That's because you then fall in line behind everyone who wanted your second choice school as thief first choice. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.

#### **Immediate Acceptance: Midstream Reversal**



#### 8th-graders' shot at elite high schools better Chicago Sun-Times November 12, 2009

Poring over data about eighth-graders who applied to the city's elite college preps, Chicago Public Schools officials discovered an alarming pattern.

High-scoring kids were being rejected simply because of the order in which they listed their college prep preferences.

``I couldn't believe it," schools CEO Ron Huberman said. ``It's terrible."

CPS officials said Wednesday they have decided to let any eighth-grader who applied to a college prep for fall 2010 admission re-rank their preferences to better conform with a new selection system.

Previously, some eighth-graders were listing the most competitive college preps as their top choice, forgoing their chances of getting into other schools that would have accepted them if they had ranked those schools higher, an official said.

Under the new policy, Huberman said, a computer will assign applicants to the highest-ranked school they quality for on their list.

• Glenn (1991)

As an example of how school selections change, analysis of first-place preferences in Boston for sixth-grade enrollments in 1989 (the first year of controlled choice in Boston) and 1990 shows that the number of relatively popular schools doubled in only the second year of controlled choice. The strong lead of few schools was reduced as others "tried" harder

• Too optimistic?

#### 2007 Ban of Immediate Acceptance in England



- Section 2.13 of 2007 School Admissions Code
  - In setting oversubscription criteria, the admissions authorities for all maintained schools must not:
    - Give priority to children according to the order of other schools named as preferences by their parents, including `first preference first' arrangements
- Rationale:
  - First-preference first criteria made the system unnecessarily complex to parents
  - Pan London Admissions Authority adopted an 'equal preference system' in 2005' to "make the admissions system fairer" and "create a simpler system for parents"
- 47 out of 148 Local education authorities in UK were forced to stop using IA

#### Why Property #1 is Important



- Admissions already feels like a high-stakes gamble, but when the algorithm rewards dishonest ranking, it's even more true
- Strategizing is imperfect because families don't know....
  - What schools are other families are choosing
  - Whether to take a chance at a popular school or not
- These differences may exacerbate inequalities between those who have learned the rules and those who haven't
- Rewarding dishonesty undermines trust in the overall system
- Choice data not reliable for scenario planning or measuring demand patterns

#### **Other Properties: Efficiency and No Justified Complaints**

- <u>Question</u>: Can we improve the assignment of a student without harming the outcome for another student?
  - If not, we say the outcome is **efficient**
  - More formally, its Pareto efficient
- What are the efficient outcomes in this example?







#### **Efficiency vs. No Justified Complaints**

- Can a student complain about their assignment and be justified?
  - A **complaint is justified** if a student wants another choice and a lower ranked student is assigned there
- Is this outcome free of justified complaints? Student i1 <-> School s2 (first choice) Student i2 <-> School s1 (first choice) Student i3 <-> School s3 (last choice)

No!

• Student i3 prefers School s1 and has higher priority than Student i2 who is assigned there









#### Property #1: Honesty is the best policy

#### Property #2: "Assign as Many as Possible a High Choice" (Efficiency)

Property #3: "Implement District Admissions Priorities" (No justified complaints)

#### Can't have it all: No system satisfies all three properties (mathematical fact)

Among algorithms in which honesty is the best policy, a tradeoff exists between #2 and #3

## POLL

## What do you think is more important?



- 1. Efficiency = "Assign as Many as Possible a High Choice"
- 2. No justified complaints = "Implement District Admissions Priorities"
- 3. It Depends
- 4. Neither





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#### Efficient Exchange via Top Trading Cycles



• TTC assigns students as follows:

**Step 1**: Every available school points to its highest priority individual among all individuals. Every individual points to her most preferred school among all available schools. A cycle is a list of a schools and individuals such that  $s_k$  points to  $i_k$  and  $i_k$  points to  $s_{k+1}$  for every k.

For every cycle, assign each individual with the school she points to in that cycle and remove the individual and decrease the capacity of the school. If the capacity is zero, remove it along with individuals in the cycle from the problem.

**Step k**: Repeat the algorithm in the next round until no more students remain.















































$$\pi_{s_{1}}: i_{1} - i_{3} - i_{2}$$

$$\pi_{s_{2}}: i_{2} - i_{1} - i_{3}$$

$$\underbrace{\pi_{s_{3}}: i_{2} - i_{1} - i_{3}}_{\text{School Priorities}}$$

$$\mu^{\mathsf{TTC}} = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

*i*<sub>1</sub>, *i*<sub>2</sub>: first choice *i*<sub>3</sub>: third choice



#### Case study 2: New Orleans Recovery School District



- New Orleans Recovery School District was formed in 2003 to facilitate state control of schools in New Orleans; role expanded considerably following Hurricane Katrina in 2005
- In 2012, RSD became the nation's first to integrate assignment between traditional public and charter schools
  - By 2014, the district became 100% charter
- Based on past experiences in NYC, officials decided to streamline admissions with a single best offer assignment system, known as OneApp
- Early on, officials decided that all RSD schools would use sibling and walk-zone priorities, and an even number would break ties within applicants



#### Case study 2: New Orleans Recovery School District



- This algorithm is an organized procedure to find ways for mutually beneficial swaps between students, where after swaps all involved students are better off.
- In this algorithm, honesty is the best policy.
- Since we find all mutually beneficial swaps, it also produces an efficient outcome.

#### **Gale-Shapley Deferred Acceptance Algorithm**



• DA assigns students as follows:

**Step 1**: Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. Any remaining proposers are rejected.

In general, at

**Step k**: Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. Any remaining proposers are rejected.



































 $P_{i_1}: s_2 \succ s_1 \succ s_3$   $P_{i_2}: s_1 \succ s_2 \succ s_3$   $\underbrace{P_{i_3}: s_1 \succ s_2 \succ s_3}_{\text{Student Preferences}}$ 

























 $P_{i_1}: s_2 \succ s_1 \succ s_3$   $P_{i_2}: s_1 \succ s_2 \succ s_3$   $\underbrace{P_{i_3}: s_1 \succ s_2 \succ s_3}_{\text{Student Preferences}}$ 



























$$\pi_{s_1} : i_1 - i_3 - i_2 \\ \pi_{s_2} : i_2 - i_1 - i_3 \\ \underbrace{\pi_{s_3} : i_2 - i_1 - i_3}_{\text{School Priorities}}$$

$$\mu^{\mathsf{DA}} = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

 $i_1, i_2$ : second choice  $i_3$ : third choice

#### Case study 3: New York City

#### The New York Times

#### How Game Theory Helped Improve New York City's High School Application Process

#### By Tracy Tullis

Dec. 5, 2014

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Tuesday was the deadline for eighth graders in New York City to submit applications to secure a spot at one of 426 public high schools. After months of school tours and tests, auditions and interviews, 75,000 students have entrusted their choices to a computer program that will arrange their school assignments for the coming year. The weeks of research and deliberation will be reduced to a fraction of a second of mathematical calculation: In just a couple of hours, all the sorting for the Class of 2019 will be finished.

- This algorithm iteratively goes down each students rank order list, tentatively assigning students to seats.
- No student is finally accepted until there are no new rejections.
- In this algorithm, honesty is the best policy.
- The algorithm never produces a justified complaint.



#### DA vs. TTC

$$P_{i_1}: s_2 \succ s_1 \succ s_3$$

$$P_{i_2}: s_1 \succ s_2 \succ s_3$$

$$\underbrace{P_{i_3}: s_1 \succ s_2 \succ s_3}_{\text{Student Preferences}}$$

$$\mu^{\mathsf{TTC}} = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

 $i_1, i_2$ : first choice  $i_3$ : third choice

$$\pi_{s_1} : i_1 - i_3 - i_2 \\ \pi_{s_2} : i_2 - i_1 - i_3 \\ \underbrace{\pi_{s_3} : i_2 - i_1 - i_3}_{\text{School Priorities}}$$

$$\mu^{\mathsf{DA}} = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

 $i_1, i_2$ : second choice  $i_3$ : third choice

## POLL



Which algorithm do you think is better for K12 public school enrollment?

- 1. Deferred Acceptance (DA)
- 2. Top Trading Cycles (TTC)
- 3. Immediate Acceptance
- 4. None of the Above
- 5. It Depends

#### **Comparison of Outcomes: Evidence from the Field**



- In Boston and New Orleans, TTC slightly increases the number who obtain top choices
  - We know this because both cities use strategy-proof systems and we can re-run algorithms
- In New York City, TTC would have a larger effect on number who obtain top choice
   Difference likely driven by the wide-ranging school priorities in NYC vs. elsewhere
- In 2012, New Orleans moved to DA after using TTC
  - Louisiana Scholarship Program schools joined the system; eventually OPSB schools did, and they have several schools rank applicants
    - With schools that rank applicants, justified complaints may be a bigger problem
  - DA also seemed easier to explain to participants

#### Algorithms



- An algorithm is only one part of the school choice process, but it is the foundation of a well-functioning single best offer system
- Three key properties:
  - Honesty is the best policy
  - Give as many students as possible a top choice (efficiency)
  - No justified complaints
- An honest algorithm has many virtues
  - Levels the playing field, Easier to give advice, Possible to simulate scenarios
- Other criteria depend on policy aims and local context
- Today is a high-level overview but there are important details we've left for future webinars
  - Diversity policies, walk-zone reserves, family link policies



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## **Questions?**

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